Entangled Wavepackets in the Vacuum

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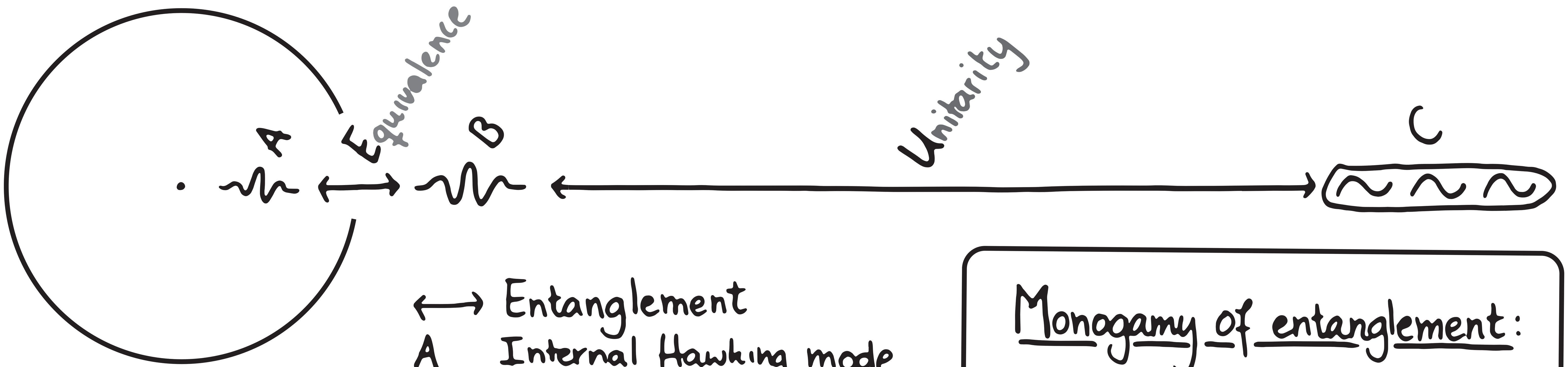
The firewall paradox



⟷ Entanglement
A Internal Hawking mode
B Outgoing Hawking mode
C Early radiation

Monogamy of entanglement:

$\neg(E \wedge U) \rightarrow$ problem!

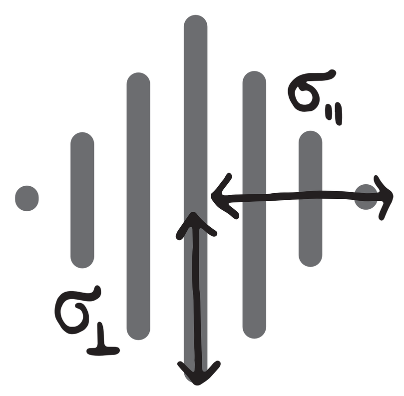
Question

How does the localization of modes influence their entanglement?

Results

In our cases of interest, the overall state is always the Minkowski vacuum. Since this is a pure state any entropy of a subsystem is due to entanglement. We calculate the von Neumann entropy S for various subsystems. Since S is a monotonically increasing function of the expectation value of the number operator $\langle \hat{N} \rangle$ we sometimes only list the latter.

Gaussian Wavepacket



$$\langle \hat{N} \rangle = \frac{n(n+2)}{64(k\sigma_\perp)^4}\left(1 + \frac{3}{(k\sigma_\parallel)^2} + \frac{45}{4(k\sigma_\parallel)^4} + \dots\right)$$

n number of perpendicular directions
k approximate momentum

Spaghetti: large $\sigma_\parallel$, small $\sigma_\perp$
Pancake : large $\sigma_\perp$, small $\sigma_\parallel$.

"In the vacuum, spaghetti has more entropy than pancakes."

Single Rindler Wavepacket.

Boring correction ↓ Interesting correction! ↓

$$\langle \hat{N} \rangle = \frac{1}{e^{2\pi k} - 1}\left(1 + \frac{f(k)}{\sigma_\parallel^2} + \frac{n}{8\sqrt{2}\,k}\frac{\sigma_\parallel}{\sigma_\perp^2}e^{\sigma_\parallel^2(1-k^2)} + \dots\right)$$

n number of perpendicular directions
k approximate Rindler-momentum
$\sigma_\parallel$ is measured in Rindler-coordinates.
f some known, boring function of k

Two Rindler wavepackets

In the limit $\sigma_\perp \rightarrow \infty$:

Some known, boring function of k ↓

$$I \equiv S_L + S_R - S_{LR} = 2\tilde{S} + \frac{g(k)}{\sigma_\parallel^2} - \frac{1 + \log(2x^2)}{x^2}$$

↑ Entropy of the pair in the limit $\sigma_\perp \rightarrow \infty$, $\sigma_\parallel \rightarrow \infty$

Two competing effects!

$x = 2\sinh(\pi k)\sigma_\parallel$

In the limit $\sigma_\perp \not\rightarrow \infty$:

Work in progress

Conclusion

- It is possible to construct a pair of highly entangled, localized modes
- Localization in the $\perp$-direction can be much more important than localization in the $\parallel$-direction.
- Paradox persists